A relativistic formalism for computation of irrotational binary stars in quasi equilibrium states

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We present relativistic hydrostatic equations for obtaining irrotational binary neutron stars in quasi equilibrium states in 3+1 formalism. Equations derived here are different from those previously given by Bonazzola, Gourgoulhon, and Marck, and have a simpler and more tractable form for computation in numerical relativity. We also present hydrostatic equations for computation of equilibrium irrotational binary stars in first post-Newtonian order.

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I. INTRODUCTION

Preparation of reliable theoretical models on late inspiraling stage of binary neutron stars is one of the most important issues for gravitational wave astronomy. This is because they are one of promising sources for gravitational wave detector such as LIGO [1], VIRGO [2], GEO600 [3] and TAMA [4]. From their signals, we will get a wide variety of physical information on neutron stars such as their mass, spin, and so on if we have a theoretical template of them [5]. In particular, a signal from very late inspiraling stage just prior to merging may contain physically important information on neutron stars such as their radius [5], which will be utilized for determining equation of state of neutron stars [6].

Binary neutron stars evolve due to radiation reaction of gravitational waves, so that they never settle down to equilibrium states. However, the emission time scale will be always longer than the orbital period outside their innermost stable circular orbit (ISCO), so that we may consider that they are in quasi equilibrium states in their inspiraling phase even near ISCO. Motivated by this idea, there have been several works in which sequence of equilibrium states of binary neutron stars are computed and the sequence is regarded as an evolutionary track; for example, we have obtained corotational equilibrium states in first post-Newtonian approximation [7]; Baumgarte et al. have obtained corotational equilibrium states in a relativistic frame work using conformal flat approximation [8]. Up to now, however, all relativistic works have been done assuming corotational velocity field [9]. As pointed out previously [10], corotation is not a adequate assumption for velocity field of realistic binary neutron stars, because effect of viscosity is negligible for evolution of neutron stars in binary and as a result, their velocity field are expected to be irrotational (or nearly irrotational).

For computation of realistic quasi equilibrium states of coalescing binary neutron stars just prior to merging, Bonazzola, Gourgoulhon, and Marck (BGM) [11] recently presented a relativistic formalism. In their formulation, they assume a helicoidal Killing vector ℓ^{μ}, and then project relativistic hydrodynamic equations onto a hypersurface orthogonal to ℓ^{μ}. After that, they impose their irrotational condition on the hypersurface and derive hydrostatic equations for the irrotational fluid. We think, however, that there were several inadequate treatments in their work. First one is their definition of irrotational condition, because their irrotational condition is nothing but a necessary condition for irrotation even in the case when we assume existence of ℓ^{μ} [12]. In general case, their condition is not identical with the irrotational condition. Second, in numerical relativity, we usually solve equations such as Hamiltonian constraint, momentum constraint, and equations for gauge conditions, using spatial coordinates on the hypersurface, Σ_t, which is perpendicular to unit normal n^{ν}. Due to this reason, they had to re-project their equations onto Σ_t. As a result, their equations for determining velocity field have complicated form. Finally, in their formalism, it is necessary to solve a complicated vector Poisson equation for relativistic cases, which should be unnecessary for irrotational fluid. Although we may get correct results using their formalism, we had better obtain a simpler and more tractable formalism. The purpose in this paper is to present such one.

In section II, we derive hydrostatic equations for irrotational fluid from relativistic hydrodynamic equations. We use 3+1 formalism and project the hydrodynamic equations onto Σ_t. Then, we impose an irrotational condition on Σ_t, which agrees with the relativistic irrotational condition [12]. As a result of projection onto Σ_t, we obtain hydrostatic equations on Σ_t, and hence, they have suitable forms to be solved in numerical relativity. Also, in our formalism, we need to solve only one Poisson type equation for a scalar field for determination of vector field. In section III, taking Newtonian limit, we show that well-known Newtonian hydrostatic equations are derived from the present formalism. In section IV, we give first post-Newtonian hydrostatic equations for irrotational fluid as well as gravitational potentials to be

solved. Section V is devoted to summary. Throughout this paper, c denotes speed of light, and we use units in which gravitational constant is unity. We use units $c = 1$ in section II for convenience and recover c in sections III and IV. Latin and Greek indices denote three dimensional (3D) spatial components $(1-3)$ and four dimensional (4D) components $(0-3)$, respectively. As spatial coordinates, we use the Cartesian coordinates $x^k = (x^1, x^2, x^3)$.

II. RELATIVISTIC FLUID EQUATIONS IN 3+1 FORMALISM

Since we use 3+1 formalism in general relativity, we write the line element as

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= (-\alpha^2 + \beta_k\beta^k)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j, \end{aligned} \tag{2.1}$$

where $g_{\mu\nu}$, α, $\beta_i = \gamma_{ij}\beta^j$ and γ_{ij} are 4D metric, the lapse function, shift vector, and 3D spatial metric respectively. Using the unit normal to 3D spatial hypersurface Σ_t,

$$n^\mu = \left(\frac{1}{\alpha}, -\frac{\beta^i}{\alpha}\right) \quad \text{and} \quad n_\mu = \Big(-\alpha, 0, 0, 0\Big), \tag{2.2}$$

γ_{ij} is written as

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu. \tag{2.3}$$

Hereafter, we use ∇_μ and D_i as the covariant derivatives with respect to $g_{\mu\nu}$ and γ_{ij}, respectively.

We assume the energy momentum tensor of the perfect fluid as

$$T^{\mu\nu} = \rho\left[1 + \varepsilon + \frac{P}{\rho}\right]u^\mu u^\nu + Pg^{\mu\nu}, \tag{2.4}$$

where ρ, ε, P, and u^μ denote the rest mass density, specific internal energy, pressure, and four velocity, respectively. We assume polytropic equation of state $P = (\Gamma - 1)\rho\varepsilon$, where $\Gamma = 1 + 1/n$ and n is the polytropic index. From adiabatic condition, we also get $P = K\rho^\Gamma$, where K is a constant. For the following, we define h as

$$\begin{aligned} h &= 1 + \varepsilon + \frac{P}{\rho} \\ &= 1 + \frac{K\Gamma}{\Gamma - 1}\rho^{\Gamma-1} = 1 + \int\frac{dP}{\rho}. \end{aligned} \tag{2.5}$$

From the conservation equation for the energy momentum tensor,

$$\nabla_\mu T^\mu_{\ \nu} = 0, \tag{2.6}$$

we get the hydrodynamic equation as

$$u^\mu\nabla_\mu\tilde{u}_\nu + \nabla_\nu h = 0, \tag{2.7}$$

where $\tilde{u}_\nu = hu_\nu$, and we use the conservation equation for rest mass density as

$$\nabla_\mu(\rho u^\mu) = 0. \tag{2.8}$$

To rewrite the hydrodynamic equation, we decompose u^μ as

$$u^\mu = u^0(\ell^\mu + V^\mu), \tag{2.9}$$

and assume that (1) ℓ^μ is a timelike vector of its component $(1, \ell^i)$, and (2) V^μ is a spatial vector, $V^\mu n_\mu = 0$, i.e., $V^\mu = (0, V^i)$. By using ℓ^μ and V^μ, we get the following relations;

$$\begin{aligned} \gamma_i^{\ \nu}\ell^\mu\nabla_\mu\tilde{u}_\nu &= \gamma_i^{\ \nu}\Big[\mathcal{L}_\ell\tilde{u}_\nu - \tilde{u}_\mu\nabla_\nu\ell^\mu\Big] \\ &= \gamma_i^{\ \nu}\left[\mathcal{L}_\ell\tilde{u}_\nu - \tilde{u}_\mu\nabla_\nu\left(\frac{u^\mu}{u^0} - V^\mu\right)\right] \\ &= \gamma_i^{\ \nu}\left[\mathcal{L}_\ell\tilde{u}_\nu + h\nabla_\nu\left(\frac{1}{u^0}\right) + \tilde{u}_\mu\nabla_\nu V^\mu\right] \\ &= \gamma_i^{\ \nu}\mathcal{L}_\ell\tilde{u}_\nu + hD_i\left(\frac{1}{u^0}\right) + {}^{(3)}\tilde{u}_k D_i V^k \\ &\quad + n_\sigma\tilde{u}^\sigma\gamma_i^{\ \nu}V^\mu\nabla_\nu n_\mu, \end{aligned} \tag{2.10}$$

and

$$\gamma_i^{\ \nu}V^\mu\nabla_\mu\tilde{u}_\nu = V^k D_k{}^{(3)}\tilde{u}_i - n_\sigma\tilde{u}^\sigma\gamma_i^{\ \nu}V^\mu\nabla_\mu n_\nu, \tag{2.11}$$

where $\mathcal{L}_\ell$ denotes the Lie derivative with respect to ℓ^μ and ${}^{(3)}\tilde{u}_i$ is a spatial vector defined as $\gamma_i^{\ k}\tilde{u}_k$. Using these relations, projection of Eq. (2.7) onto the 3D hypersurface Σ_t becomes

$$\begin{aligned} u^0&\left[\gamma_i^{\ \nu}\mathcal{L}_\ell\tilde{u}_\nu + V^k D_k{}^{(3)}\tilde{u}_i + {}^{(3)}\tilde{u}_k D_i V^k + hD_i\left(\frac{1}{u^0}\right)\right] \\ &+ D_i h = 0. \end{aligned} \tag{2.12}$$

We can rewrite this equation as

$$\begin{aligned} \gamma_i^{\ \nu}\mathcal{L}_\ell\tilde{u}_\nu &+ D_i\left(\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k\right) \\ &+ V^k(D_k{}^{(3)}\tilde{u}_i - D_i{}^{(3)}\tilde{u}_k) = 0. \end{aligned} \tag{2.13}$$

Besides the conservation equation of the energy momentum tensor, we have the conservation equation for rest mass density (2.8). We note that for the case of barotropic equation of state such as $P = K\rho^\Gamma$, Eq. (2.8) is also derived from the conservation equation of the energy momentum tensor. This implies that if we solve Eq. (2.8), we do not have to take into account n^μ component of Eq. (2.7). Using Eq. (2.9), Eq. (2.8) is written as

$$\alpha[\mathcal{L}_\ell(\rho u^0) + \rho u^0\nabla_\mu\ell^\mu] + D_i(\rho\alpha u^0 V^i) = 0. \tag{2.14}$$

Now, we assume that ℓ^μ is a Killing vector such that $\nabla_\mu\ell_\nu + \nabla_\nu\ell_\mu = 0$, $\mathcal{L}_\ell\tilde{u}_\nu = 0$ and $\mathcal{L}_\ell(\rho u^0) = 0$, and we write its component as $(1, -\Omega x^2, \Omega x^1, 0)$, where Ω is identified with the orbital angular velocity with respect to distant inertial observer. We note that fluid exists inside the light cylinder $|x^k| \ll c\Omega^{-1}$, and existence of the

Killing vector is assumed within it. We also note that ℓ^μ defined here is identical with the helicoidal Killing vector defined by BGM [11]. If the Killing vector exists, we can derive hydrostatic equations for two interesting cases. One is the corotational case where we simply set $V^i = 0$. Then, we get a well-known result as [13]

$$\frac{h}{u^0} = \text{constant}, \tag{2.15}$$

and continuity equation is trivially satisfied in this case.

The other is the case where ${}^{(3)}\tilde{u}_i$ satisfies an "irrotational condition" defined as

$$W_{ij} \equiv D_i {}^{(3)}\tilde{u}_j - D_j {}^{(3)}\tilde{u}_i = 0, \tag{2.16}$$

and hence

$${}^{(3)}\tilde{u}_i = D_i\phi, \tag{2.17}$$

where ϕ is a scalar field. Then, the hydrodynamic equation (2.13) is integrated to give

$$\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k = \text{constant}. \tag{2.18}$$

Note that V^k and u^0 are written as

$$V^k = -\ell^k - \beta^k + \frac{1}{hu^0}\gamma^{kl}D_l\phi, \tag{2.19}$$

$$u^0 = \frac{1}{\alpha}\left[1 + h^{-2}\gamma^{kl}D_k\phi D_l\phi\right]^{1/2}, \tag{2.20}$$

so that we can rewrite left-hand side of Eq. (2.18) as

$$\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k = h\alpha^2 u^0 - (\ell^k + \beta^k){}^{(3)}\tilde{u}_k. \tag{2.21}$$

By substituting Eq. (2.19) into Eq. (2.14), we get a Poisson type equation for determining ϕ as

$$D_i(\rho\alpha h^{-1}D^i\phi) - D_i\{\rho\alpha u^0(\ell^i + \beta^i)\} = 0. \tag{2.22}$$

Hence, hydrodynamic equations which should be solved for determination of equilibrium states reduce to only two hydrostatic equations (2.18) and (2.22). We do not have to solve any equations for vector potentials which were introduced in the formalism of BGM [11].

We note that definition of irrotation in the 4D covariant form should be [12]

$$\begin{aligned}\omega_{\mu\nu} &= P_\sigma{}^\mu P_\lambda{}^\nu(\nabla_\mu u_\nu - \nabla_\nu u_\mu) \\ &= h^{-1}(\nabla_\mu\tilde{u}_\nu - \nabla_\nu\tilde{u}_\mu) = 0,\end{aligned} \tag{2.23}$$

where $P_\sigma{}^\mu = g_\sigma{}^\mu + u_\sigma u^\mu$, and we use Eq. (2.7) to rewrite the first line into the second line. When $\omega_{\mu\nu}$ is vanishing initially for a fluid element, it remains zero along the trajectory of the fluid element for the perfect fluid [12]. Hence, $\omega_{\mu\nu} = 0$ is just the irrotational condition. In our present irrotational condition (2.16), Eq. (2.23) is satisfied on the 3D hypersurface Σ_t trivially. However, it is not trivial whether or not projection of Eq. (2.23) to $n^\mu\gamma^\nu{}_k$ component is satisfied. (Projection to $n^\mu n^\nu$ component is trivially satisfied.) We here show that it is really guaranteed due to Eq. (2.16). By operating $n^\mu\gamma^\nu{}_k$ to $\nabla_\mu\tilde{u}_\nu - \nabla_\nu\tilde{u}_\mu$, we get

$$\begin{aligned}&n^\mu\gamma^\nu{}_k(\nabla_\mu\tilde{u}_\nu - \nabla_\nu\tilde{u}_\mu) \\ &= \gamma^\nu{}_k\mathcal{L}_n\tilde{u}_\nu - \gamma^\nu{}_k(\tilde{u}_\mu\nabla_\nu n^\mu + n^\mu\nabla_\nu\tilde{u}_\mu) \\ &= \gamma^\nu{}_k\mathcal{L}_n\tilde{u}_\nu - D_k(n^\mu\tilde{u}_\mu) \\ &= \gamma^\nu{}_k\mathcal{L}_n\tilde{u}_\nu + D_k(h\alpha u^0) \equiv W_k,\end{aligned} \tag{2.24}$$

where we use $n^\mu\tilde{u}_\mu = -h\alpha u^0$. From a straightforward calculation, we can rewrite $\gamma^\nu{}_k\mathcal{L}_n\tilde{u}_\nu$ as

$$\begin{aligned}\gamma^\nu{}_k\mathcal{L}_n\tilde{u}_\nu = \frac{1}{\alpha}\Big[&\gamma^\nu{}_k\mathcal{L}_\ell\tilde{u}_\nu + h\alpha u^0 D_k\alpha - (\beta^j + \ell^j)\partial_j {}^{(3)}\tilde{u}_k \\ &- {}^{(3)}\tilde{u}_j\partial_k(\beta^j + \ell^j)\Big],\end{aligned} \tag{2.25}$$

where ∂_k denotes partial derivative on Σ_t. Hence,

$$\begin{aligned}W_k = \frac{1}{\alpha}\Big[&\gamma^\nu{}_k\mathcal{L}_\ell\tilde{u}_\nu - (\beta^j + \ell^j)\partial_j {}^{(3)}\tilde{u}_k \\ &- {}^{(3)}\tilde{u}_j\partial_k(\beta^j + \ell^j) + \partial_k(h\alpha^2 u^0)\Big].\end{aligned} \tag{2.26}$$

Using the hydrodynamic equation (2.13) and an identity (2.21), we obtain

$$\begin{aligned}W_k &= \frac{1}{\alpha}(V^j + \beta^j + \ell^j)(-\partial_j {}^{(3)}\tilde{u}_k + \partial_k {}^{(3)}\tilde{u}_j) \\ &= \frac{1}{\alpha}(V^j + \beta^j + \ell^j)W_{kj}.\end{aligned} \tag{2.27}$$

Eq. (2.27) implies that $W_k = 0$ if Eq. (2.16) is satisfied. Note that to derive Eq. (2.27) we have not assumed the fact that ℓ^μ is a Killing vector. Therefore, Eq. (2.16) is the necessary and sufficient condition for the irrotational condition in general case. Note that Eq. (2.18) itself does not mean irrotation in general. Even for the case when a Killing vector ℓ^μ exists, it is nothing but a necessary condition for irrotation.

III. NEWTONIAN LIMIT

In the Newtonian limit, metric variables can be expanded as

$$\alpha = 1 - \frac{U}{c^2} + O(c^{-4}), \tag{3.1}$$

$$\beta^k = O(c^{-3}), \tag{3.2}$$

$$\gamma_{ij} = \delta_{ij} + O(c^{-2}), \tag{3.3}$$

where U denotes the Newtonian potential which satisfies

$$\Delta U = -4\pi\rho, \tag{3.4}$$

and Δ is the flat Laplacian. By using $v^i \equiv u^i/u^0$, components of u^μ which we need here is also expanded as

$$u^0 = 1 + \frac{1}{c^2}\left\{\frac{1}{2}v^2 + U\right\} + O(c^{-4}), \tag{3.5}$$

$$u^i = u_i = \frac{v^i}{c} + O(c^{-3}), \tag{3.6}$$

where $v^2 = \sum_i v^i v^i$. Note also $\ell^\mu = (1, \ell^i/c)$ and $V^\mu = (0, V^i/c)$. For the corotational case ($V^i = 0$), we get Newtonian limit of left-hand side of Eq. (2.15) as

$$\left[\frac{h}{u^0}\right]_{\text{full rela.}} \longrightarrow 1 + \frac{1}{c^2}\left[-\frac{v^2}{2} - U + \int\frac{dP}{\rho}\right]. \tag{3.7}$$

Since $V^k = 0$, v^k is equal to ℓ^k, and $v^2 = R^2\Omega^2$ where $R^2 = (x^1)^2 + (x^2)^2$. Substituting this relation of v^2 into Eq. (3.7), we get a well-known result

$$-\frac{R^2\Omega^2}{2} - U + \int\frac{dP}{\rho} = \text{constant}. \tag{3.8}$$

For the irrotational case, Newtonian limit of the left-hand side of Eq. (2.18) becomes

$$\left[\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k\right]_{\text{full rela.}} \longrightarrow$$
$$1 + \frac{1}{c^2}\left[-\frac{v^2}{2} - U + \int\frac{dP}{\rho} + \sum_k v^k(-\ell^k + v^k)\right]. \tag{3.9}$$

In Newtonian order, $v^k = \partial_k\phi_{\rm N}$, where ϕ is expanded as $\phi_{\rm N}/c + O(c^{-3})$. So that we get

$$\frac{1}{2}\sum_k(\partial_k\phi_{\rm N})^2 - U + \int\frac{dP}{\rho} - \sum_k \ell^k\partial_k\phi_{\rm N} = \text{constant}. \tag{3.10}$$

Eq. (3.10) agrees with that of BGM [11].

From continuity equations in Newtonian order, we obtain equations for $\phi_{\rm N}$ as

$$\rho\Delta\phi_{\rm N} + \sum_k(\partial_k\phi_{\rm N} - \ell^k)\partial_k\rho = 0. \tag{3.11}$$

Eq. (3.11) is solved under boundary condition,

$$\sum_k(\partial_k\phi_{\rm N} - \ell^k)\partial_k\rho = 0, \tag{3.12}$$

at stellar surface.

IV. FIRST POST-NEWTONIAN EQUATIONS

In this section, we derive hydrostatic equations in first post-Newtonian order. The equations for the corotational case agree with those shown in previous papers [14] [7], so that we here derive equations only for the irrotational case. In first post-Newtonian approximation, metric in the standard post-Newtonian gauge can be expanded as [15]

$$\alpha = 1 - \frac{U}{c^2} + \frac{1}{c^4}\left[\frac{U^2}{2} + X\right] + O(c^{-6}), \tag{4.1}$$

$$\beta^k = \frac{1}{c^3}\hat{\beta}_k + O(c^{-5}), \tag{4.2}$$

$$\gamma_{ij} = \delta_{ij}\left[1 + \frac{2}{c^2}U\right] + O(c^{-4}), \tag{4.3}$$

where X and $\hat{\beta}_k$ are obtained from

$$\Delta X = 4\pi\rho\left(2U + 2\sum_k(\partial_k\phi_{\rm N})^2 + \varepsilon + \frac{3P}{\rho}\right), \tag{4.4}$$

$$\hat{\beta}_k = -\frac{7}{2}P_k + \frac{1}{2}\left(\partial_k\chi + \sum_j x^j\partial_k P_j\right), \tag{4.5}$$

and

$$\Delta P_k = -4\pi\rho\partial_k\phi_{\rm N}, \tag{4.6}$$

$$\Delta\chi = 4\pi\rho\sum_k(\partial_k\phi_{\rm N})x^k. \tag{4.7}$$

Note that to derive these Poisson equations, we use a relation in Newtonian order, $v^k = \partial_k\phi_{\rm N}$.

Using a post-Newtonian relation,

$$\alpha u^0 = 1 + \frac{1}{2c^2}\sum_k(\partial_k\phi_{\rm N})^2 + \frac{1}{c^4}\left[-\frac{1}{8}\left(\sum_k(\partial_k\phi_{\rm N})^2\right)^2 + \sum_k\partial_k\phi_{\rm N}\partial_k\phi_{\rm PN} - (\eta + U)\sum_k(\partial_k\phi_{\rm N})^2\right] + O(c^{-6}), \tag{4.8}$$

where we expand ϕ as $\phi_{\rm N}/c + \phi_{\rm PN}/c^3 + O(c^{-5})$ and $\eta = \varepsilon + P/\rho$, first post-Newtonian expansion of Eq. (2.18) becomes

$$\left[\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k\right]_{\text{full rela.}} \longrightarrow$$
$$1 + \frac{1}{c^2}\left[\eta - U + \frac{1}{2}\sum_k(\partial_k\phi_{\rm N})^2 - \sum_k\ell^k\partial_k\phi_{\rm N}\right] + \frac{1}{c^4}\left[-\eta U + \frac{1}{2}U^2 + X - \frac{1}{2}(\eta + 3U)\sum_k(\partial_k\phi_{\rm N})^2 - \frac{1}{8}\left(\sum_k(\partial_k\phi_{\rm N})^2\right)^2 + \sum_k\partial_k\phi_{\rm N}\partial_k\phi_{\rm PN} - \sum_k\ell^k\partial_k\phi_{\rm PN} - \sum_k\hat{\beta}_k\partial_k\phi_{\rm N}\right]$$
$$= \text{constant}. \tag{4.9}$$

First post-Newtonian expansion of continuity equation is also derived as

$$\sum_i \partial_i(\rho A_i) = 0, \tag{4.10}$$

where

$$A_i = -\ell^i + \partial_i\phi_{\rm N} + \frac{1}{c^2}\biggl\{-\ell^i\biggl(\frac{1}{2}\sum_k(\partial_k\phi_{\rm N})^2 + 3U\biggr) -\eta\partial_i\phi_{\rm N} - \hat{\beta}_i + \partial_i\phi_{\rm PN}\biggr\}. \tag{4.11}$$

If $\phi_{\rm N}$ is obtained from Eq. (3.11), Eq. (4.10) is regarded as equation for $\phi_{\rm PN}$ and solved under boundary condition

$$\sum_i A_i\partial_i\rho = 0 \tag{4.12}$$

at stellar surface.

Eqs. (4.9) and (4.10) with Poisson equtions (4.4), (4.6), (4.7) and (3.4) are basic equations for computation of irrotational equilibrium states in first post-Newtonian order.

V. SUMMARY

In this paper, we have derived relativistic hydrostatic equations for obtaining irrotational (quasi) equilibrium configurations of binary neutron stars using 3+1 formalism. In order to derive the hydrostatic equations, we first projected hydrodynamic equations onto Σ_t and then impose the irrotational condition to obtain the hydrostatic equations. As a result, the hydrostatic equations obtained are simple and suitable for numerical relativity, compared with a previous formalism [11]. Also, as a natural consequence, in our formalism there is no vector Poisson equation to be solved and only a scalar Poisson equation is needed to be solved for determination of velocity field not only for Newtonian, but also for relativistic cases.

We also give hydrostatic equations as well as Poisson equations for gravitational potentials needed for computation of irrotational equilirium states in first post-Newtonian approximation. We think that as a first step toward fully relativistic study, we had better construct post-Newtonian configurations for firm investigation of relativistic effect on binary neutron stars. In reality, we have been able to obtain much information on relativistic effect in binary neutron stars from first post-Newtonian studies [7] [16] [17]. Up to now, however, our attention was only paid to corotational binary neutron stars. The present formalism makes it possible to extend previous studies to the irrotational case. As a first work, we plan to obtain incompressible, irrotational equilibrium states of binary stars as we carried out for the corotational case previously [16].

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[1] A. Abramovici et al. Science **256**, 325 (1992).
[2] C. Bradaschia, et al. Nucl. Instrum. and Methods **A289**, 518 (1990).
[3] J. Hough, in *Proceedings of the Sixth Marcel Grossmann Meeting*, edited by H. Sato and T. Nakamura (World Scientific, Singapore, 1992), p.192.
[4] K. Kuroda et al., in *Proceedings of the international conference on gravitational waves: Sources and Detectors*, edited by I. Ciufolini and F. Fidecard(World Scientific, 1997), p.100.
[5] For example, K. S. Thorne, in *Proceeding of Snowmass 94 Summer Study on Particle and Nuclear Astrophysics and Cosmology*, eds. E. W. Kolb and R. Peccei (World Scientific, Singapore, 1995) and references cited therein.
[6] L. Lindblom, Astrophys. J. **398**, 569 (1992).
[7] M. Shibata, Phys. Rev. D **55**, 6019 (1997); Prog. Theor. Phys. **96**, 317 (1996); M. Shibata, K. Taniguchi, and T. Nakamura, Prog. Theor. Phys. supplement **128**, 295 (1997).
[8] T. W. Baumgarte, G. B. Cook, M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. Lett. **79**, 1182 (1997); gr-qc/9709026.
[9] However, for Newtonian case, following researchers have obtained irrotational Newtonian equilibrium states;
K. Uryu and Y. Eriguchi, astro-ph/9712203;
S. Bonazzola. E. Gourgoulhon, J.-A. Marck, talk at GRG 15, India (1997).
[10] C. S. Kochanek, Astrophys. J. **398**, 234(1992);
L. Bildsten and C. Cutler, Astrophys. J. **400**, 175(1992).
[11] S. Bonazzola. E. Gourgoulhon, J.-A. Marck, Phys. Rev. D **56**, 7740 (1997).
[12] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time* (Cambridge University Press, 1973), 84.
[13] A. P. Lightman, W. H. Press, R. H. Price and S. A Teukolsky, *Problem Book in Relativity and Gravitation* (Princeton Unversity Press, 1975), problem 16.17.
[14] S. Chandrasekhar, Astrophys. J. **148**, 621 (1967).
[15] S. Chandrasekhar, Astrophys. J. **142**, 1488 (1965):
L. Blanchet, T. Damour, and G. Schäfer, Mon. Not. R. astr. Soc. **242**, 289 (1990).
[16] K. Taniguchi and M. Shibata, Phys. Rev. D **56**, 798 (1997).
[17] M. Shibata and K. Taniguchi Phys. Rev. D **56**, 811 (1997).